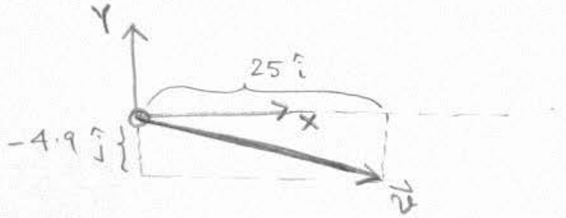
PHY 121: MECHANICS: MID TERM EXAMINATION.

SOLUTIONS

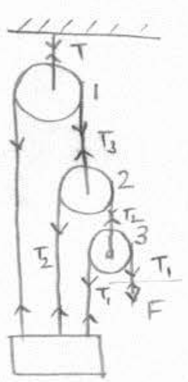
(1) $\vec{v} = 25\,\hat{i} - 4.9\,\hat{j}$ (in meters/sec)

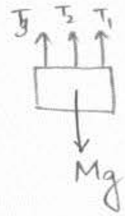


Since, the y-component of velocity is $-4.9\,\hat{j}$ (i.e. downward velocity) it has already reached its highest point of trajectory

[At highest point : $v_y = 0$
no y component of velocity]

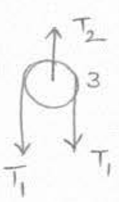
(2)





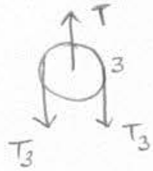
$M = 6.40$ kg

$T_1 + T_2 + T_3 = Mg$ —(i)



$T_2 = 2T_1$ (ii)

$T_3 = 2T_2$ —(iii)



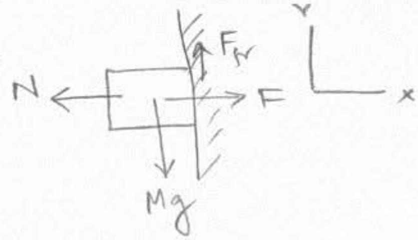
$T = 2T_3$ —(iv)

$\therefore\ T_3 = \frac{T}{2}$; $T_2 = \frac{T_3}{2} = \frac{T}{4}$; $T_1 = \frac{T_2}{2} = \frac{T}{8}$

$\therefore$ in eqn (i) $\frac{T}{8} + \frac{T}{4} + \frac{T}{2} = Mg \Rightarrow \frac{T}{8}(1+2+4) = Mg$

$\boxed{T = \frac{8}{7}Mg} \Rightarrow T = \left(\frac{8}{7}\ 6.40 \times 9.8\right)N = \underline{71.7\,N}$

(3)



$F = N$

$Mg = F_{fr}$ (for no motion)

$F_{fr\,max} = \mu_s N = 0.60 \times 12\ N = 7.2\ N$

[Maximum force of friction]

$Mg = 5\ N$ (a) Now, $\because$ $F_{fr\,max} > Mg$ The block will not move

and $F_{fr} = F_{applied} = Mg = 5\ N$.

(b) $\vec{F}'$ = force on block from the wall

$= \vec{F}_{fr} + \vec{N} = 5\hat{j} + (-12\hat{i}) = \underline{(-12\hat{i} + 5\hat{j})\,N}$ Answer

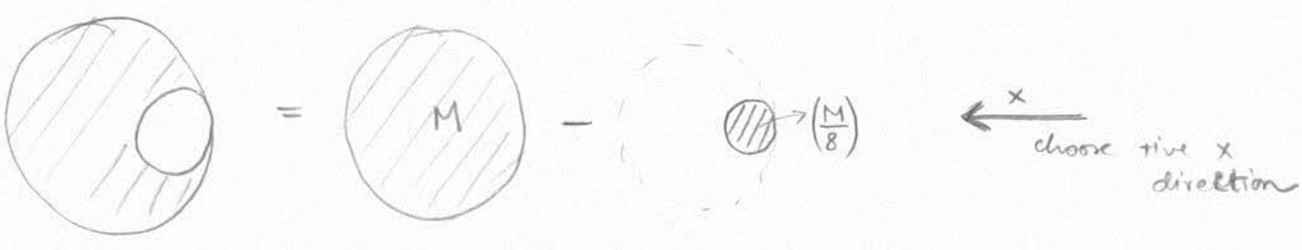
(4)

(a) $M' = M - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho$ $\qquad \rho = \dfrac{M}{\frac{4}{3}\pi R^3}$ (density)

$$M' = M - \frac{4}{3}\pi\frac{R^3}{8}\frac{M}{\frac{4}{3}\pi R^3} = \frac{7M}{8}$$

Mass of hollowed sphere $= \dfrac{7M}{8}$

(b)



$$\vec{F}_{TOT} = \vec{F}_M - \vec{F}_{M/8} = G\frac{Mm}{d^2}\hat{i} - G\left(\frac{M}{8}\right)\frac{m}{\left(d-\frac{R}{2}\right)^2}\hat{i}$$

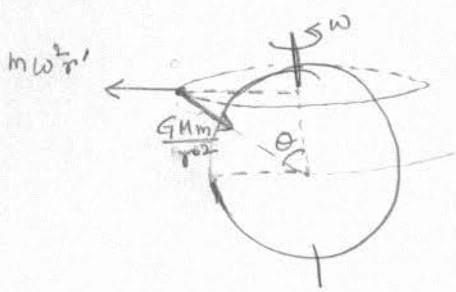
$$\vec{F}_{TOT} = GMm\left[\frac{1}{d^2} - \frac{1}{8\left(d-\frac{R}{2}\right)^2}\right]\hat{i}$$

(In +ive x direction
∴ attractive force)

(c) $$\vec{E} = \frac{\vec{F}_{TOT}}{m} = GM\left[\frac{1}{d^2} - \frac{1}{8\left(d-\frac{R}{2}\right)^2}\right]\hat{i}$$

(field is the force per unit mass)

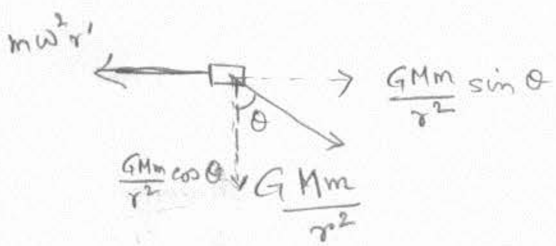
(5)



$r' = R\sin\theta$

$r = R + h$

R = radius of Earth
M = mass of Earth
m = mass of satellite
h = ht above ground of satellite.

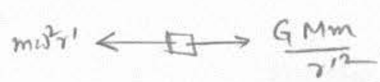


The satellite can remain only over the equator, because if $\theta \neq \frac{\pi}{2}$

$\frac{GMm}{r^2}\cos\theta$ is the unbalanced component that tries to bring it to the $\theta = \frac{\pi}{2}$

for equilibrium above the equator:

$$m\omega^2 r' = \frac{GMm}{r'^2} \qquad r' = R + h$$



$$r'^3 = \frac{GM}{\omega^2} \Rightarrow r' = R + h = \sqrt{\frac{GM}{\omega^2}} \Rightarrow \boxed{h = \sqrt{\frac{GM}{\omega^2}} - R}$$

Answer.

(6) $v_1 = 10$ km/hr $\qquad v_2 = 20$ km/hr with initial mass m_1 & m_2 respectively

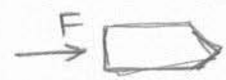
r = rate = 1000 kg/min

Let Δt be the time of crossing, time for which coal was shoveled.

Momentum equation

(a) fast barge

$$m_2 v_2 = (m_2 + \underbrace{r\Delta t}_{\text{total mass of coal transferred}}) v_2 - F\Delta t.$$



$$\therefore \quad r \cancel{\Delta t}\, v_2 = F \cancel{\Delta t} \quad \Rightarrow \quad F = r v_2 = \frac{1000}{60} \times 20 \times \frac{5}{18} = 92.3\,N$$

extra force on fast barge

this force should speed up the faster barge

(b) slow barge:

$$m_1 v_1 = (m_1 - r\Delta t) v_1 - F\Delta t$$

$$\therefore \quad -r \cancel{\Delta t}\, v_1 = F \cancel{\Delta t} \quad \Rightarrow \quad F = -r v_1$$

slowing down force on slower barge

$$F = -\frac{1000}{60} \times 10 \times \frac{5}{18} = \underline{46.3\ N}.$$

(7)

$m_1 = 2\text{ kg}$ $\quad$ $m_2 = ?$

v_{1i} $\quad$ $v_{2i} = 0$

$v_{1f} = \frac{1}{4} v_{1i}$

Momentum cons:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} = m_1 \frac{1}{4} v_{1i} + m_2 v_{2f}$$

$$\frac{3}{4} m_1 v_{1i} = m_2 v_{2f} \quad \text{—(i)}$$

K. Energy cons.

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

$$m_1 v_{1i}^2 = m_1 \left(\frac{v_{1i}}{4}\right)^2 + m_2 v_{2f}^2 \Rightarrow m_1 v_{1i}^2 \left(1 - \frac{1}{16}\right) = m_2 v_{2f}^2$$

$$m_1 v_{1i}^2 \left(\frac{15}{16}\right) = m_2 v_{2f}^2 \quad \text{—(ii)}$$

Square (i) we get : $\frac{9}{16} m_1^2 v_{1i}^2 = m_2^2 v_{2f}^2 \quad$ —(iii)

divide (iii) by (ii) : $\frac{9}{\cancel{16}} m_1 \left(\frac{\cancel{16}}{15}\right) = m_2$

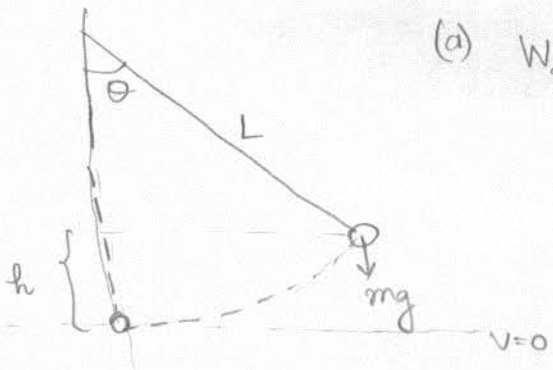
(a) $m_2 = \frac{9}{15} m_1 = \frac{9}{15} \cdot 2\text{ kg} = \underline{1.2\text{ kg}}$ Answer.

(b) if $v_{1i} = 4.0$ m/s.

$$v_{CM\ initial} = \frac{m_1 v_{1i} + m_2 \cancel{v_{2i}}^{\,0}}{m_1 + m_2} = \frac{m_1 v_{1i}}{m_1 + m_2} = \frac{2 \times 4}{(2 + 1.2)} \text{ m/s.}$$

$$v_{CM\ initial} = v_{CM\ final} = \underline{2.5\text{ m/s}}$$ Answer.

(8)



(a) $W_{gravity} = mgh$

$= mg\,(L - L\cos\theta)$

$W_{grav} = mgL\,(1-\cos\theta)$

(b) $\Delta V = -mgL\,(1-\cos\theta)$ [When the mass m comes down potential of system is lowered $\therefore \Delta V$ is -ive]

(c) $\underset{\theta=0}{V} = 0$ (at lowest point)

$\underset{\theta=\theta}{V} = mgL\,(1-\cos\theta)$ <u>Answer</u>.

(d) Answer to (a) is increased $\because$ $\theta \uparrow$ $\cos\theta \downarrow$ $\therefore W_{grav} = mgL\,(1-\cos\theta) \uparrow$

(b) is decreasing [with greater θ The change is -ive and greater in magnitude $\therefore$ decreasing ΔV]

(c) is is increasing.